# KoAT: An Automatic Complexity Analysis Tool for Integer Programs

## Workshop on Termination 2023

Nils Lommen, Eleanore Meyer, and Jürgen Giesl

# Motivation

**Goal**: Infer (upper) runtime bounds for “real-world” programs

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- Does this program terminate?



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  - Solution: Use `KoAT`!
  - Open-source complexity analysis tool for **I**nteger **T**ransition **S**ystems

# Overview

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# General Architecture

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C Integer

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$$
\begin{tikzcd}
 & & & \text{Upper Bound} \\
\text{C Integer} \arrow[r, "\texttt{clang}"] & \text{LLVM} \arrow[r, bend left, "\texttt{llvm2kittel}"] \arrow[r, bend right, "\texttt{AProVE}"'] & \text{ITS} \arrow[ur] \arrow[dr] \arrow[r, phantom, "\texttt{KoAT}"] & \phantom{x} \\
 & & & \text{Termination}
\end{tikzcd}
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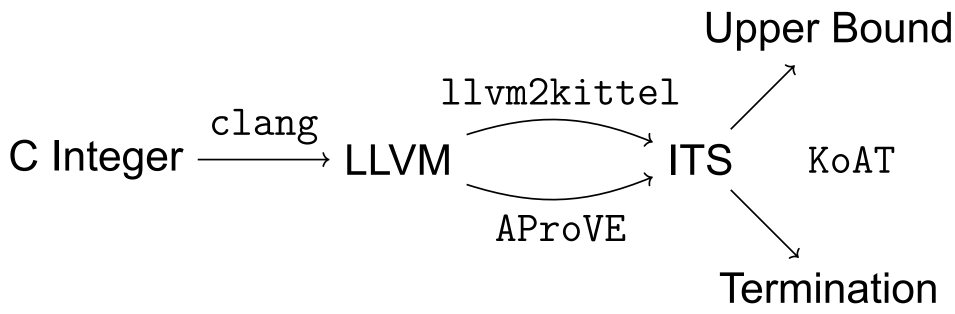


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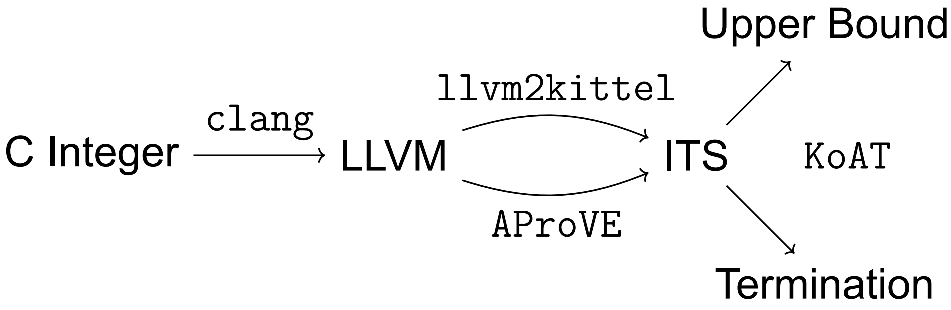


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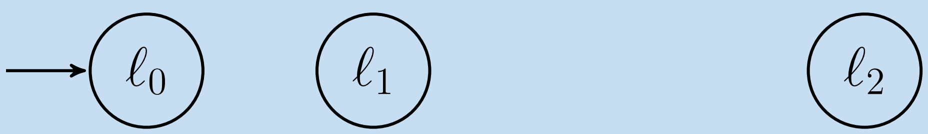

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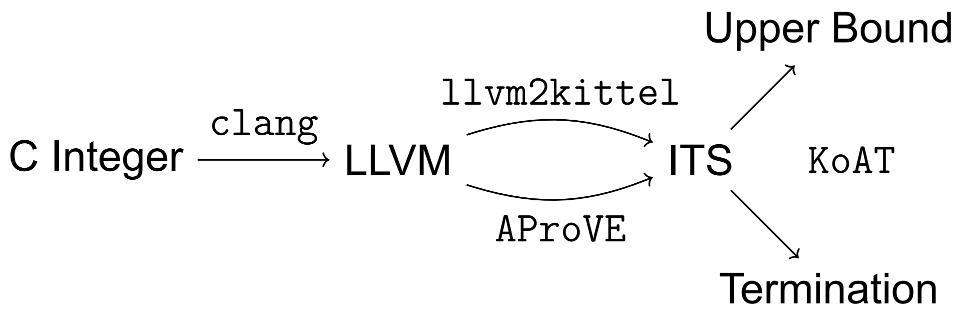


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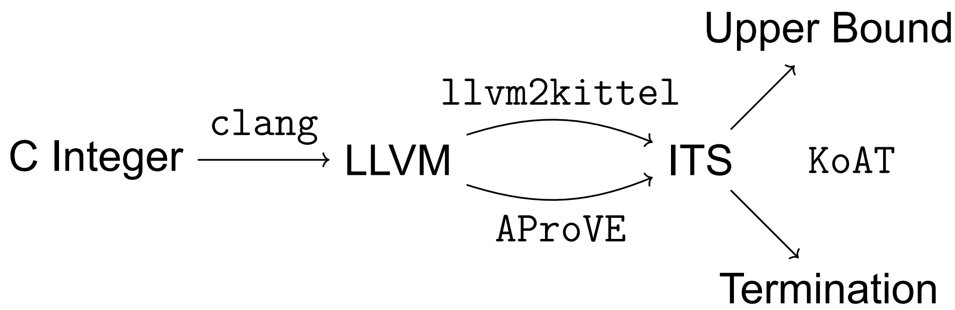


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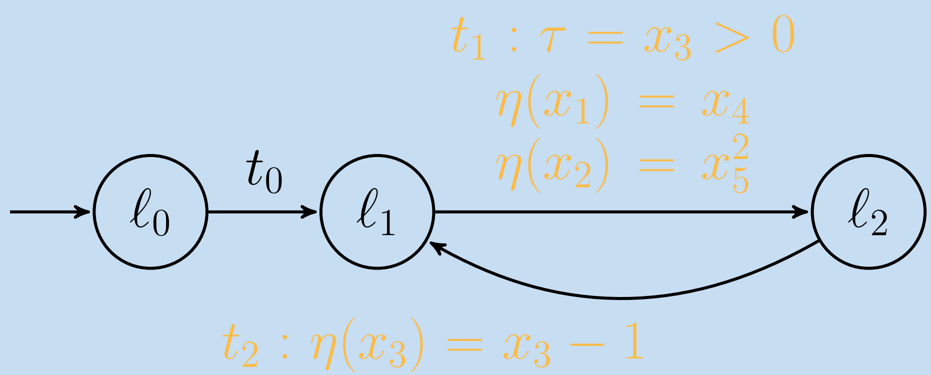



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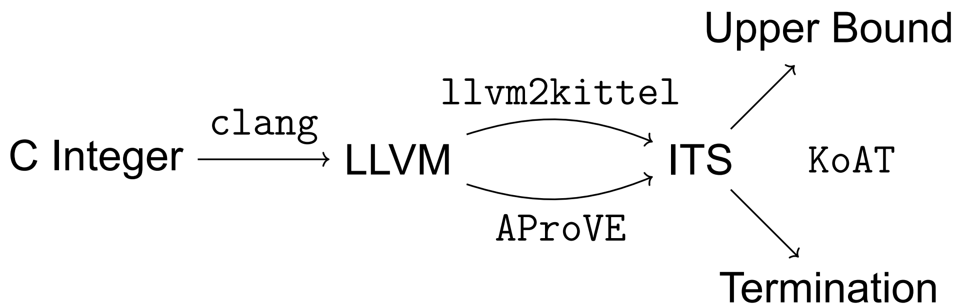


while $(x_3 > 0)$ do

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while $(x_1^2 < x_2)$ do

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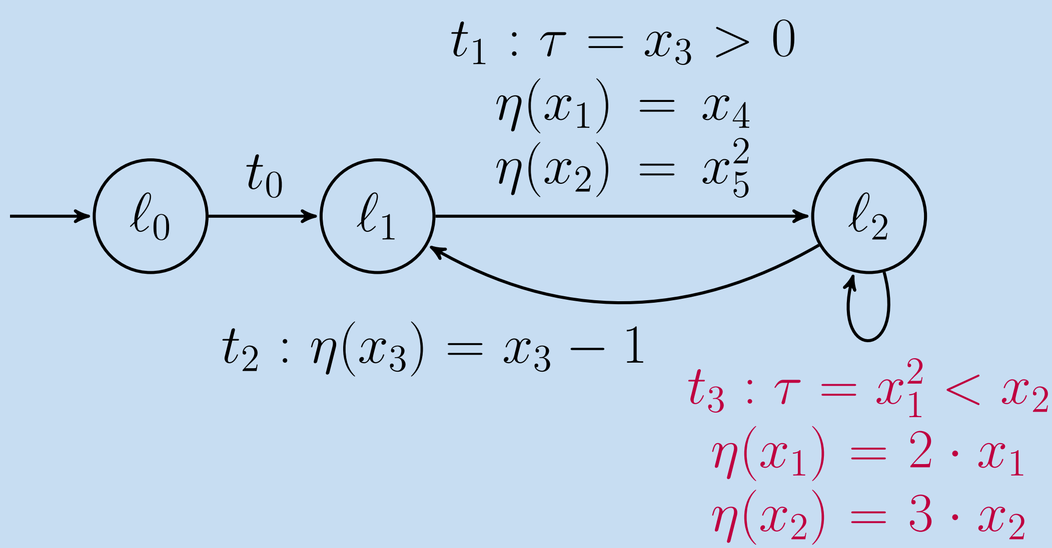

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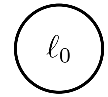

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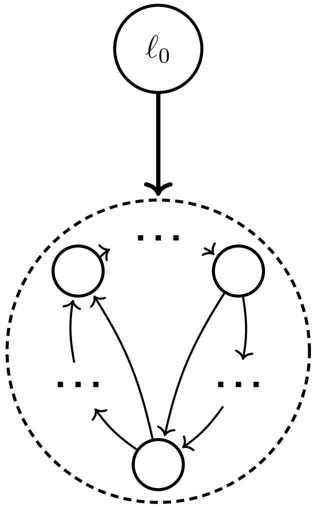



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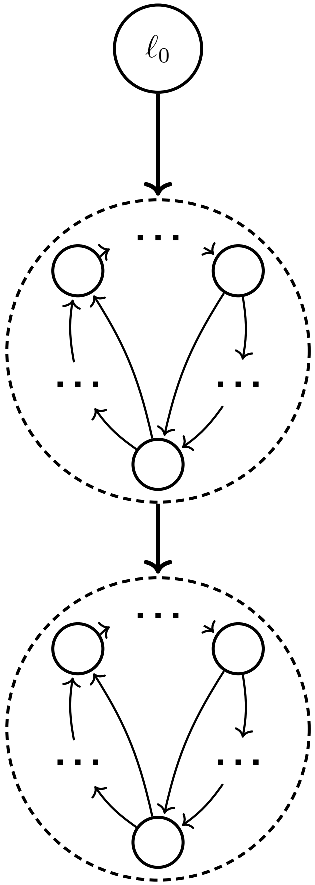



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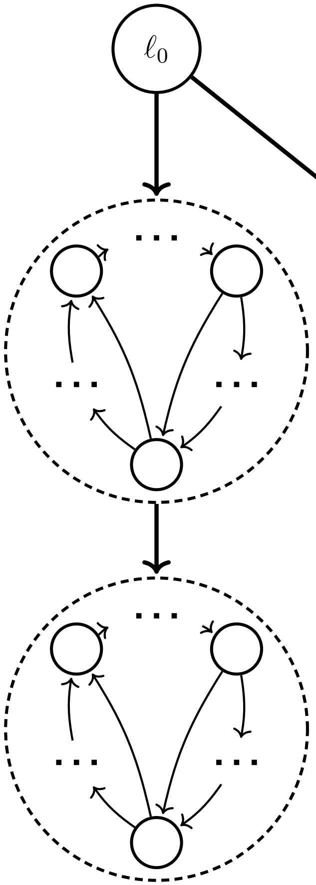




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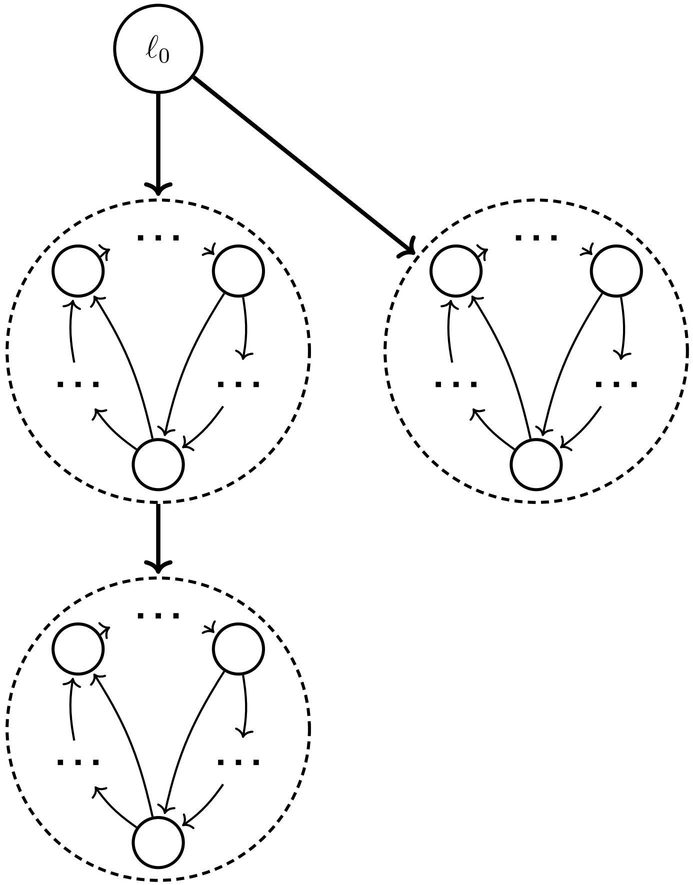




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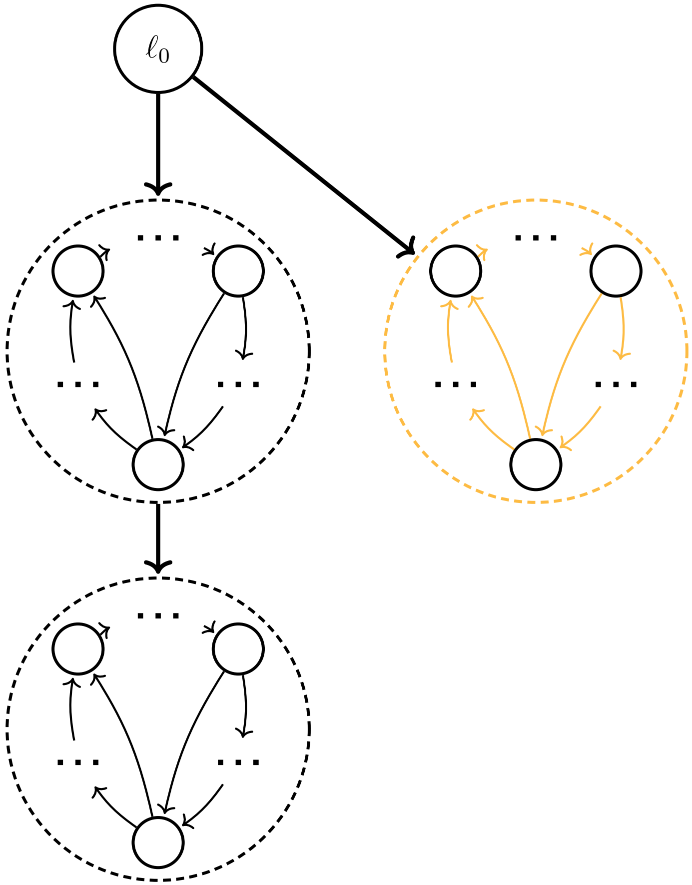



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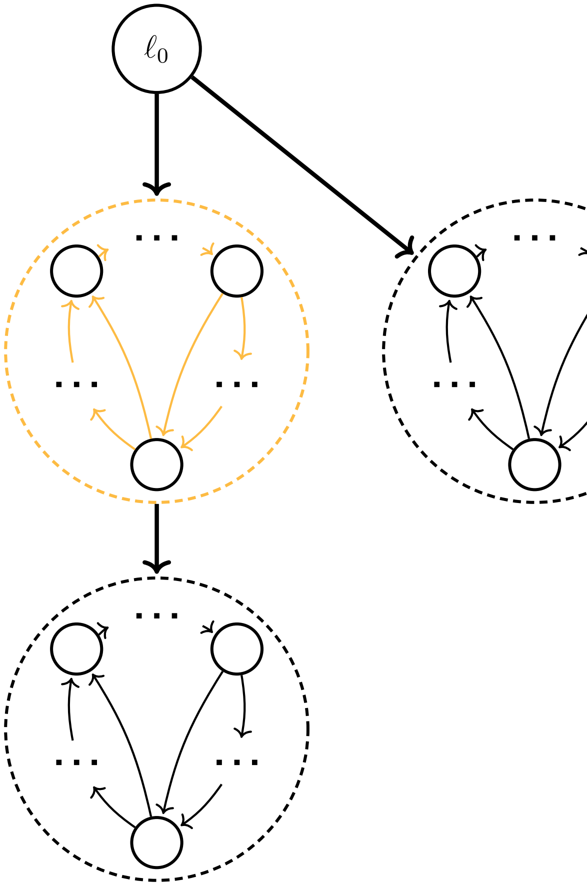



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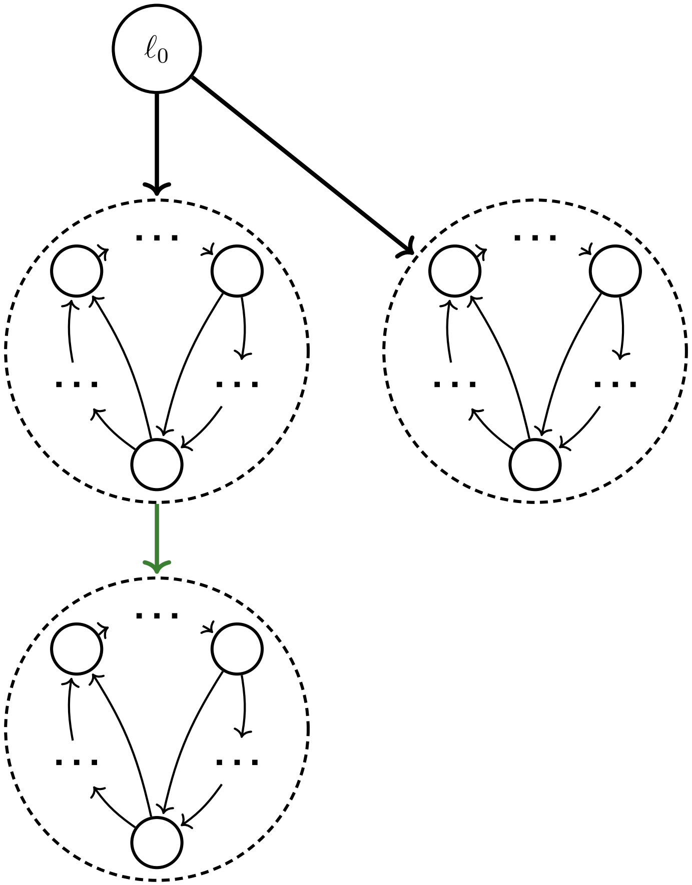




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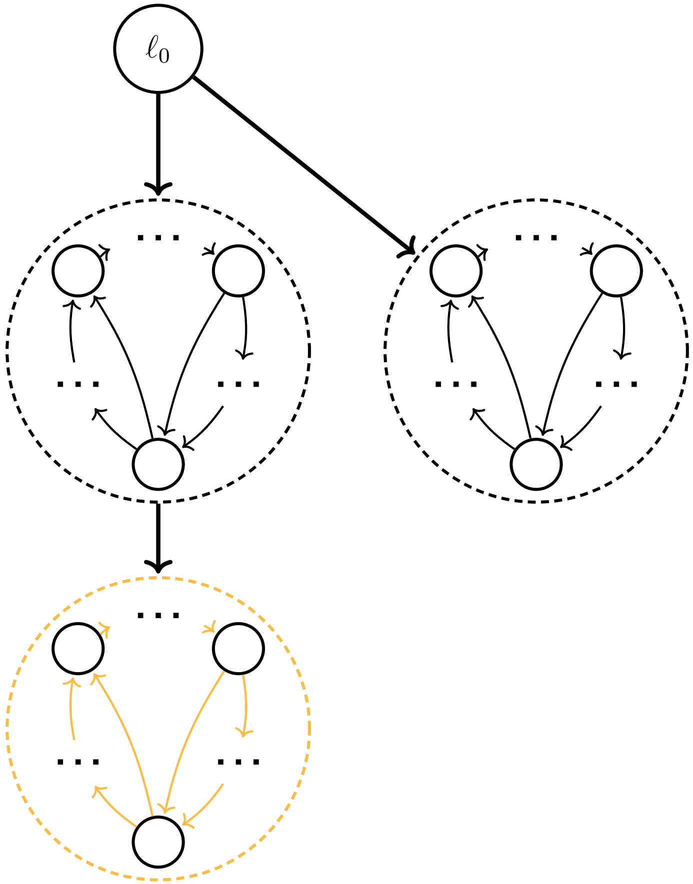




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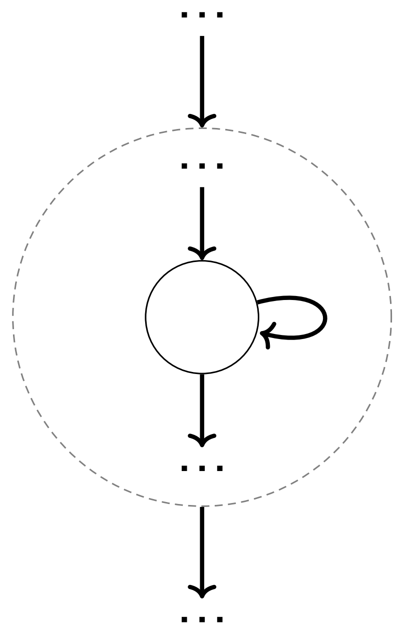
# General Architecture – Analyzing an SCC

- Alternatingly compute time and size bounds



# General Architecture – Analyzing an SCC

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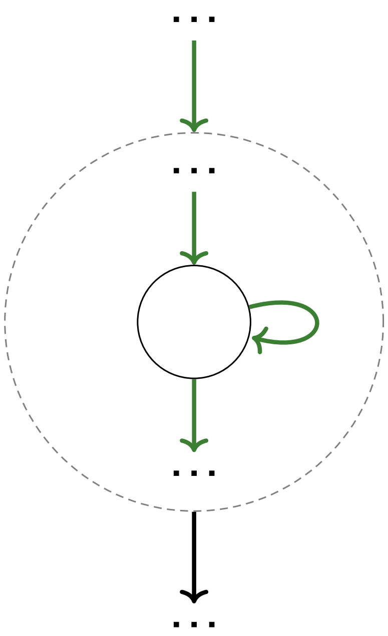
  - Compute initial global size bounds





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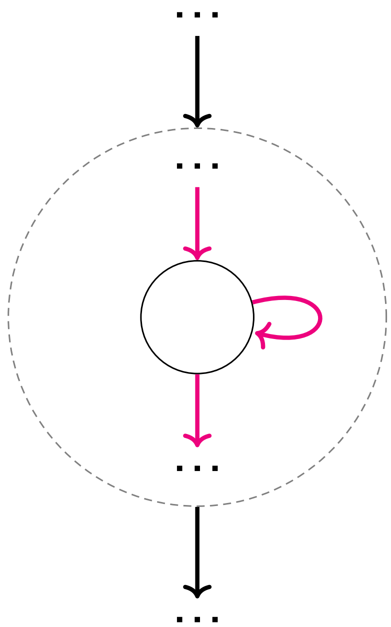



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# General Architecture – Analyzing an SCC

- Alternatingly compute time and size bounds
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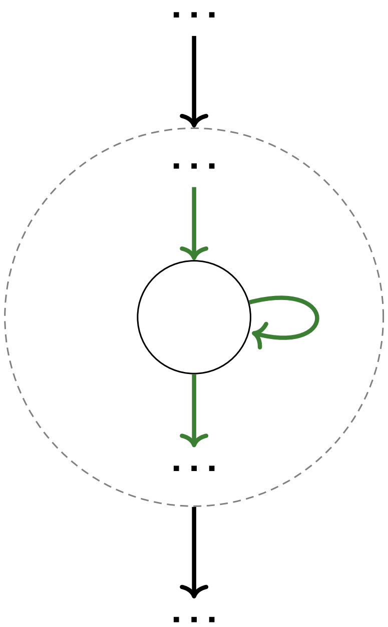






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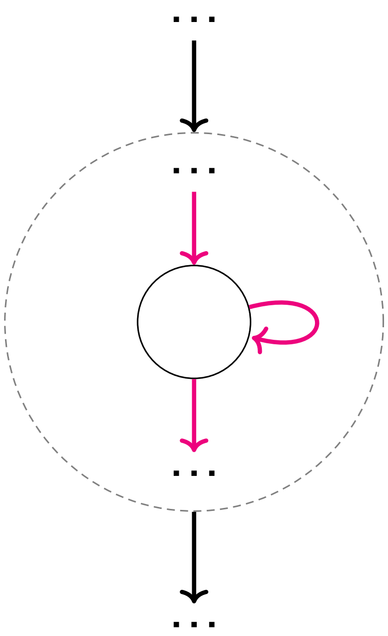



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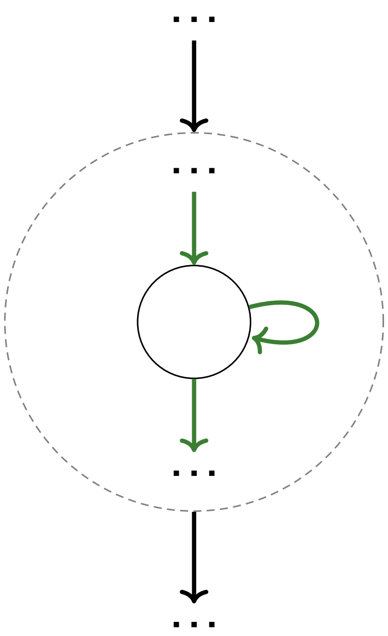



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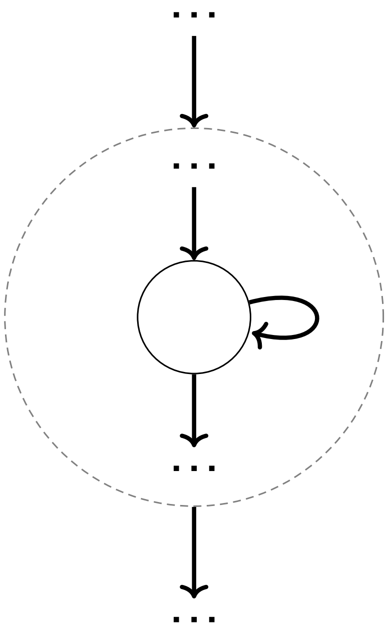




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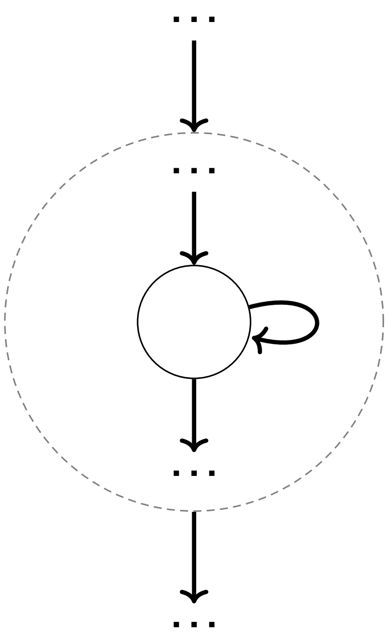



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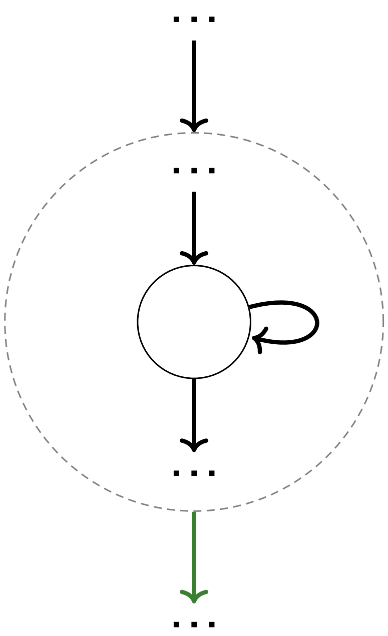




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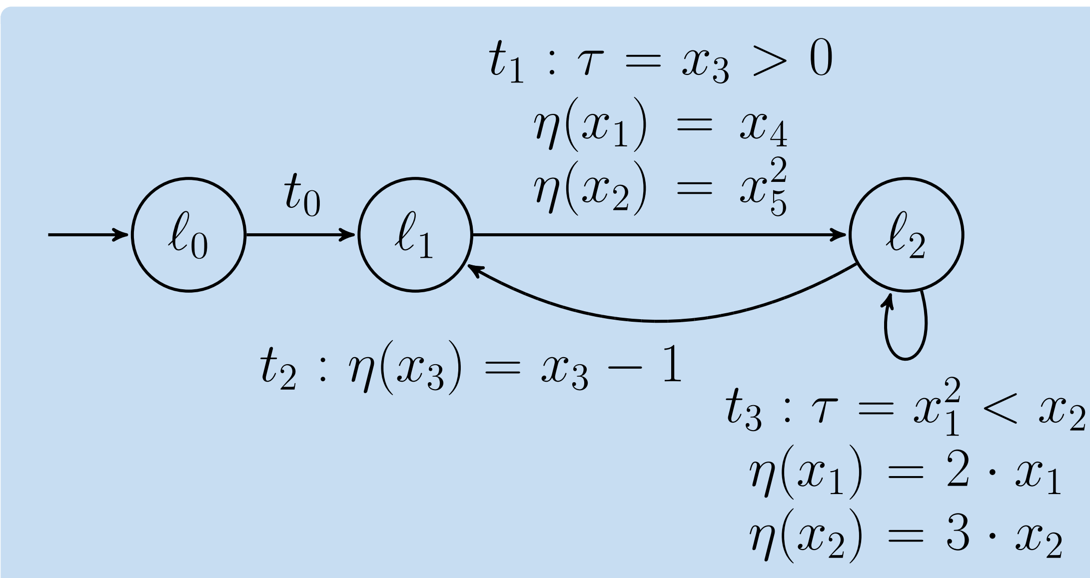




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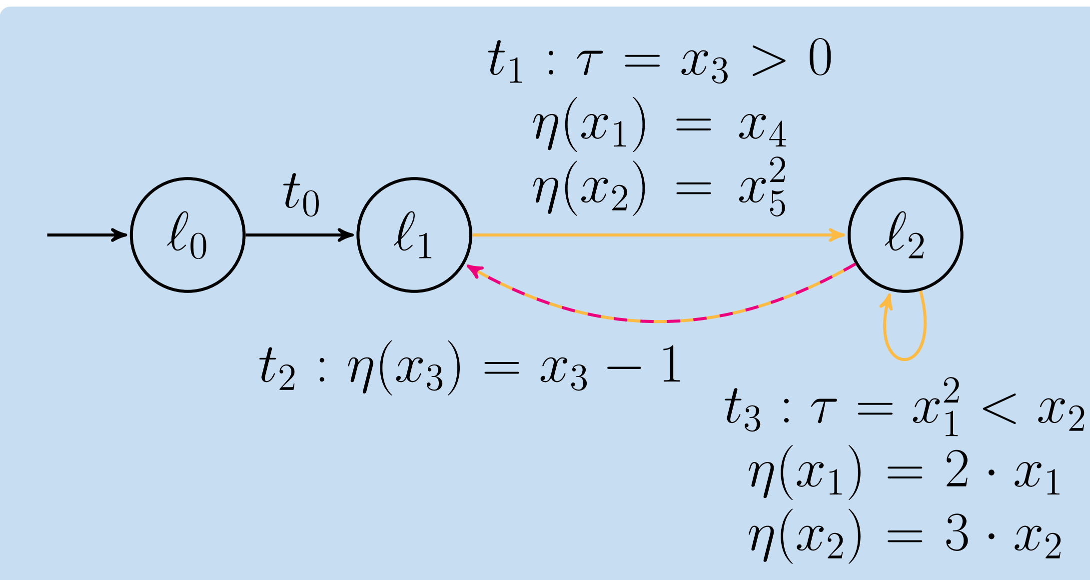


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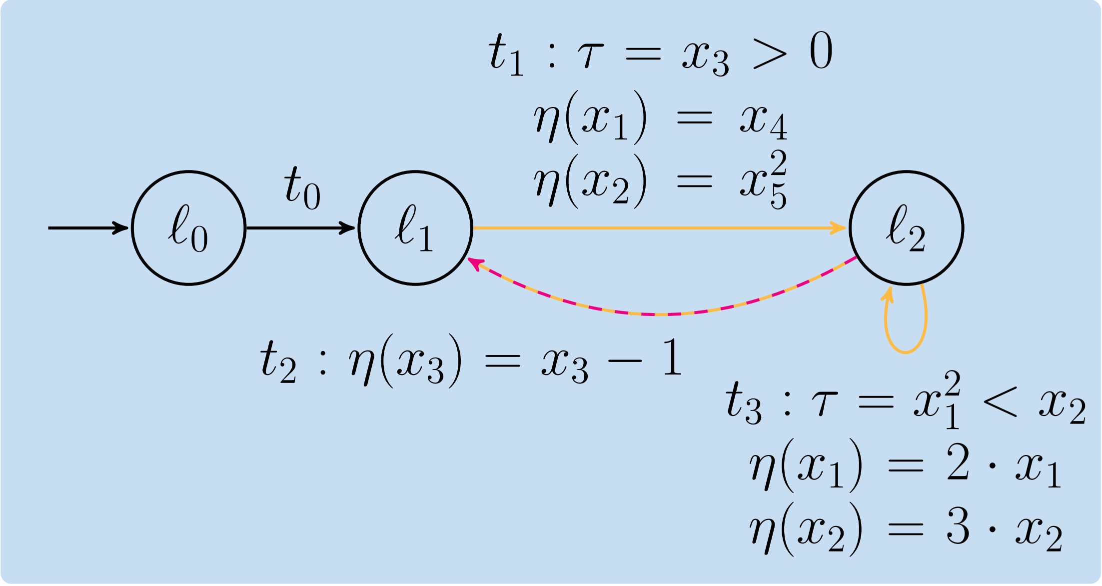


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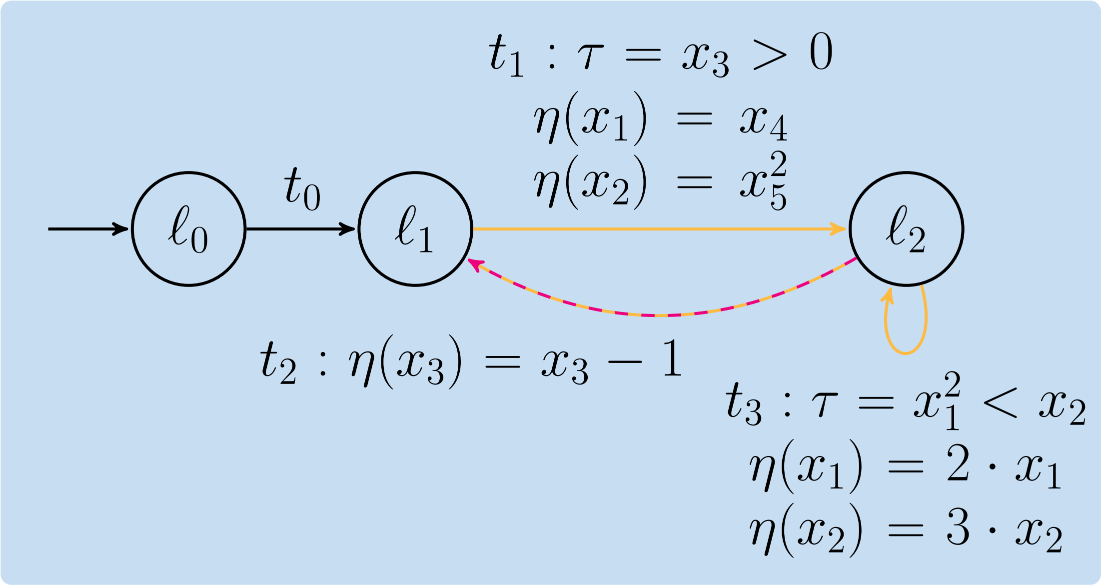


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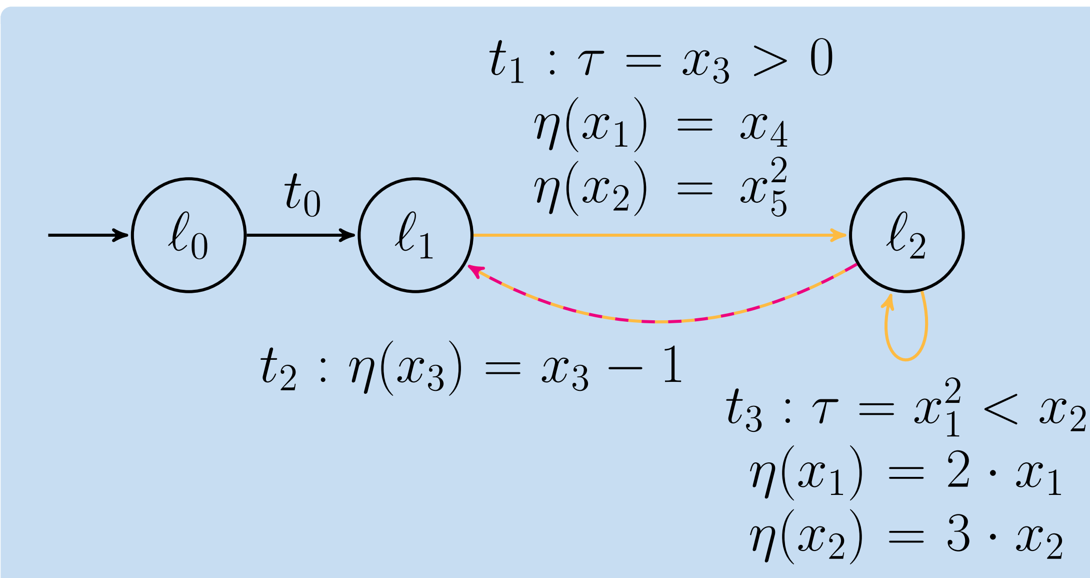


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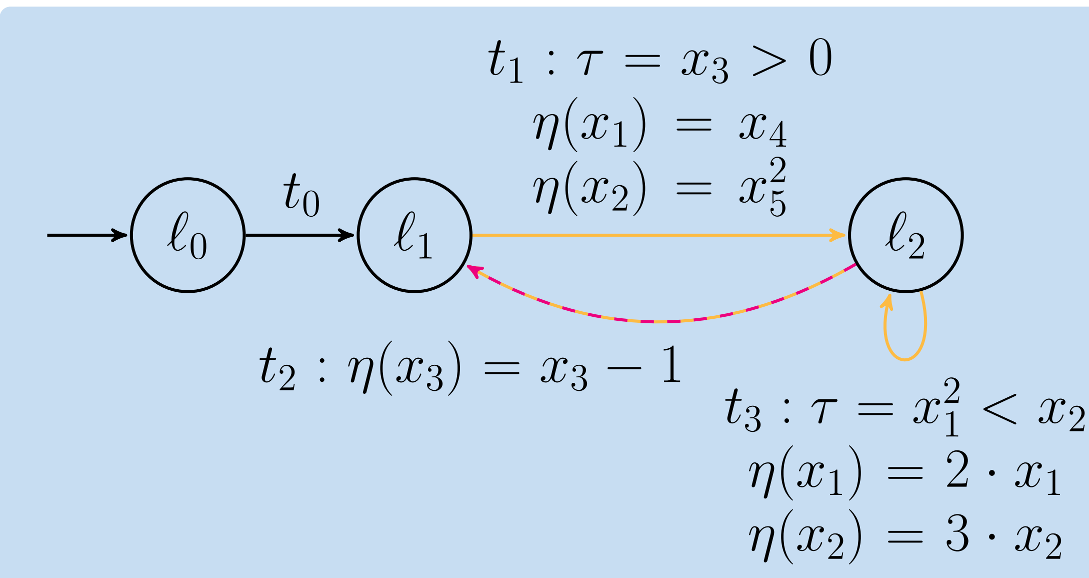


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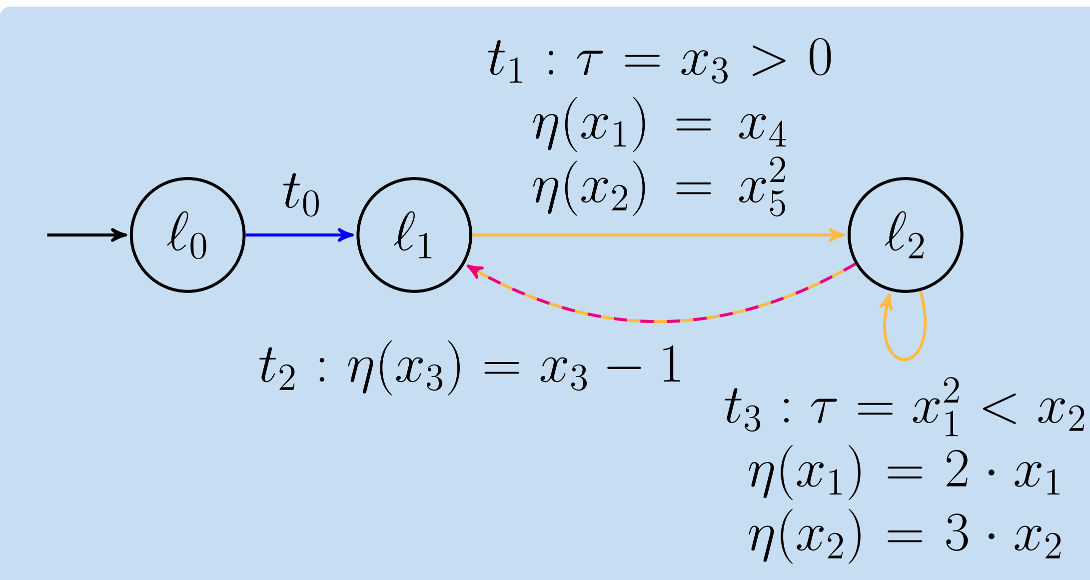


- $t_< = t_2$ and $\mathcal{T} = \{t_1, t_2, t_3\}$
- ranking function $x_3$ yields well-founded order on $\mathbb{N}$
- consider entry transition $t_0$


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# Analyzing Programs: Size Bounds

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$$\texttt{while } (x_1 > 0) \texttt{ do}$$
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leftarrow \begin{bmatrix} x_1 - 1 \\ x_2 + x_1^2 \end{bmatrix}$$
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- Closed form:
  $\mathtt{cl}_{x_2}^{n} = x_2 + n \cdot (\frac{1}{6} + x_1 + x_1^2 - x_1 \cdot n - \frac{n}{2} + \frac{n^2}{3})$


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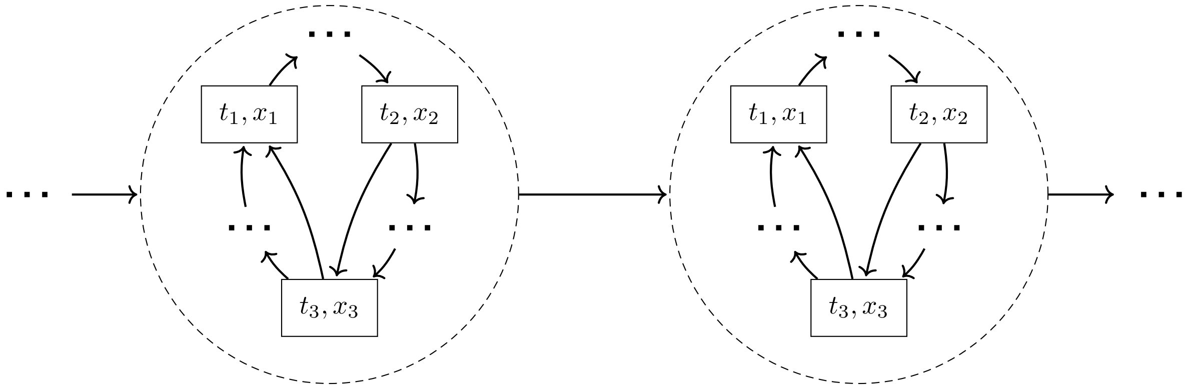




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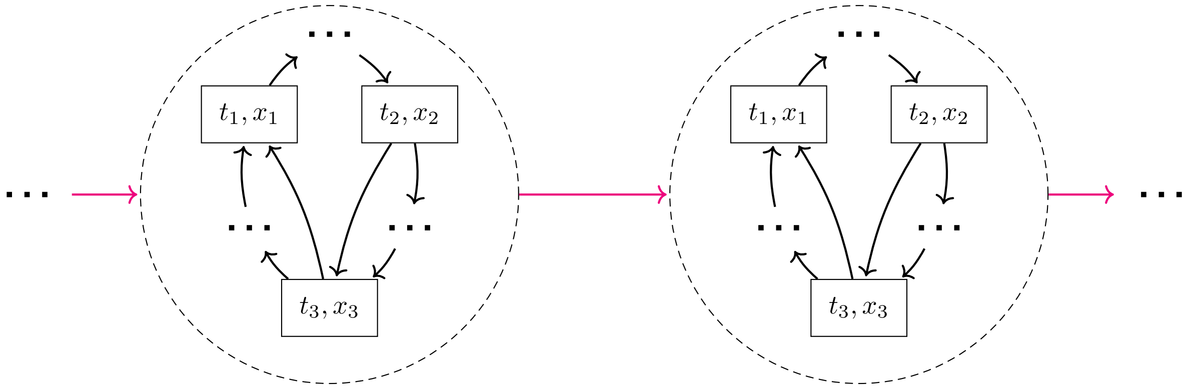




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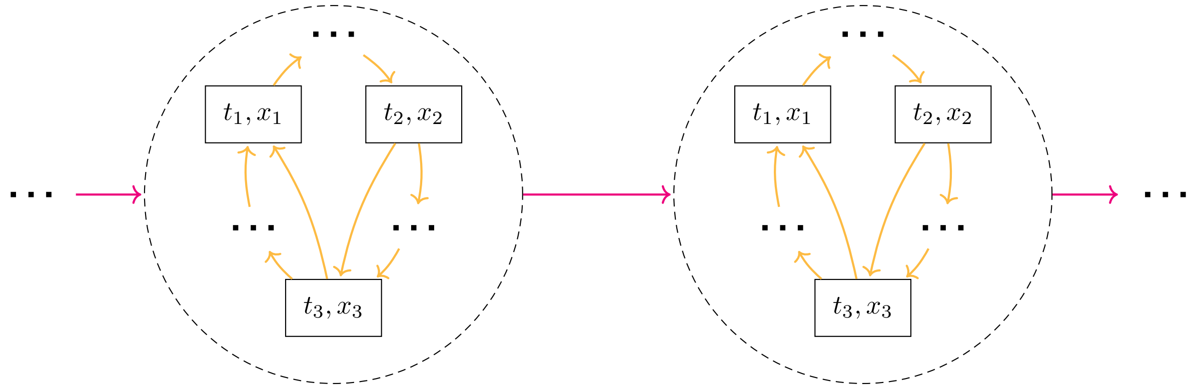

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- **Problem**: complex, nested loops

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**Control-Flow Refinement by Partial Evaluation (CFR)** [Doménech et al. '19]


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**Control-Flow Refinement by Partial Evaluation (CFR)** [Doménech et al. ’19]

- sort out certain program paths


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# Integrating Control-Flow Refinement

- **Problem**: complex, nested loops
- Loop consists of *two* phases:
  1. **then**-case is repeated until $y \leq 0$
  2. **else**-case is repeated until $x \leq 0$

  $\Rightarrow$ No run, where second phase is executed before first phase

```
while (x > 0) do
  if (y > 0) then
    y ← y − x
  else
    x ← x − 1
end
```

$\rightsquigarrow$

```
while (x > 0 ∧ y > 0) do
  y ← y − x
end
while (x > 0 ∧ y ≤ 0) do
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- CFR *modular* for SCCs with “problematic” transitions on-demand [RH ’22]

# Live-Demo

```
(GOAL COMPLEXITY)
(STARTTERM (FUNCTIONSYMBOLS l0))
(VAR A B C D E)
(RULES
  l0(A,B,C,D,E) -> l1(A,B,C,D,E) :|: C > 0
  l1(A,B,C,D,E) -> l2(D,E^2,C,D,E) :|: C > 0
  l2(A,B,C,D,E) -> l2(2*A,3*B,C,D,E) :|: A^2 < B && A > 0
  l2(A,B,C,D,E) -> l1(A,B,C - 1,D,E)
)
```

`https://koat.verify.rwth-aachen.de`


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Thank You!




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